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Power and Energy Systems

Engineering Economics

Applications Examples

Chapter 2 – Financial Mathematics

Notes:

1. Cells with black characters include inputs
2. Cells with red characters include formulas
3. Some examples need for calculations the installation of Add_Ins developed by the author. See installation instruction in the file introduction.

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PK **Panos Konstantin**

Disclaimer

The Examples are solely and exlusively indented to provide support and assistance to the readers for practicing and better understanding of the theoretical part of this book.

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The author, Panos Konstantin, believes that all information and guidance provided and all calculations in these examples are correct. Nevertheless anyone using these examples should carry out their own due dilligence and appraisal of the contents.

No warranty is made nor responsibility or liability is taken or accepted by the author for adequacy, completeness or accuracy of the examples or assumptions on which they are based.

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The author does not assume any liability to anyone for any loss or damage caused by any error or omission in this work, regardless of whether such error or omission is the result of negligence or any other reason. Any and all such liability is completely disclaimed.

Proposals for improvements of the contents are welcome and will be considered in upcoming updates!

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Examples 2.1 to 2.3 are directly calculated in the textpart of the book.

EE-Ch-2_FinancialMaths_Examples.xls
Ex_2.4_CompoundingUnequalSeries

Item	Unit	Values				
Compounding period	-	year				
Interest rate per period *"i"*	-	10.0%				
Compound factor $q = 1 + i$	-	1.10				
Timing of payment	-	beginning of period				
Year	-	1	2	3	4	5
Compound periods of each payment *"t"*	-	5	4	3	2	1
Nominal values of the payments	CU	1,500	2,500	5,000	1,500	2,500
Compound amount factor "q^t"	-	1.61	1.46	1.33	1.21	1.10
Future values of the payments	CU	2,416	3,660	6,655	1,815	2,750
Future value of all payments by the end of 5th year	**CU**					**17,296**

EE-Ch-2_FinancialMaths_Examples.xls

Ex_2.5_Discount_UnequalSeries

Item	**Unit**	**Values**				
Compounding period	-	year				
Interest rate per period *"i"*	-	10.0%				
Discount factor $q = 1 + i$	-	1.10				
Timing of payment	-	End of period				
Year "*t*"	-	1	2	3	4	5
Nominal values P_i	CU	1,500	2,500	5,000	-1,500	2,500
Present value factor "$1/q^t$	-	0.91	0.83	0.75	0.68	0.62
Present values $PV_i = P_i / q^t$	CU	1,364	2,066	3,757	-1,025	1,552
Nominal value of all payments ΣP_i	CU	10,000				
Present value of all payments ΣPV_i	**CU**	**7,714**				

The Example is calcualted directly in the textpart of the book

EE-Ch-2_FinancialMaths_Examples.xls

Ex 2.7 FV funtion

Item	Due date	
	Begin	End
Interest rate	5% /a	5% /a
Period *Nper*	20 a	20 a
Payment *Pmt*	1000	1000
Present value *PV*	0	0
Due date, *type*	1	0
Future compound amount *FV*	**34,719**	**33,066**

EE-Ch-2_FinancialMaths_Examples.xls
Ex_2.8_Interest_Construction

Item	Unit	Values
Given		
CAPEX	mln US$	1000
Loan	mln US$	700
Construction time	a	5
Equal quarterly payments	mln US$/quarter	35
Payment periods	-	20
Annual interest rate	-	6 %/a
Interest rate per period	-	1.5 %/quarter
Results		
Compound amount of loan *)	mln US$	821
Interest during construction	mln US$	121
In percent of CAPEX	**-**	**12.15%**

*) by the end of the construction period calculated with MS-Excel formula *FV*

EE-Ch-2_FinancialMaths_Examples.xls
Ex_2.9_Anu_Morgage vs.maturity

Item	**Unit**	**Maturity years**		
		10 a	20 a	30 a
Principal	€	300,000	300,000	300,000
Interest rate	% / a	6%	6%	6%
Annuity	€ / a	40,760	26,155	21,795
Total repayment	**€**	**407,604**	**523,107**	**653,840**

Ex_2.10_Anu_Morgage_vs. rate

Item	Unit	Interest rate		
		6%/a	7%/a	8%/a
Principal	€	300,000	300,000	300,000
Maturity period	years	20	20	20
Annuity	€ / a	26,155	28,318	30,556
Total repayment	**€**	**523,107**	**566,358**	**611,113**

Ex_2.11_AnuCapex

CAPEX €	Discount rate -	Project life time years	Annuity factor -	Annualized CAPEX € / a
1,000,000	8.6 %/a	10	0.1531	153,087
		20	0.1064	106,442
		50	0.0874	87,413

EE-Ch-2_FinancialMaths_Examples.xls
Ex_2.12_PVesc_PersonnelCosts

Item		Unit	Values
Given			
Cost of personnel	P_0	US$ / (Pers· a)	2,500,000
Inflation rate (for info only)		-	2.0 %/a
Escalation rate of personnel costs	*j*	-	4.0 %/a
Discount rate	*i*	-	8.6 %/a
Life time of the project	*n*	a	20
Results			
Personnel costs:			
Life time cost in current US$, undiscounted		US$	50,000,000
Present value without escalation *)	*j = 0.0%*	US$	23,487,058
Present value *with escalation* *)	*j = 4.0%*	US$	32,737,690

$$PV_{n\_esc} = P_0 \cdot \sum_{t=1}^{t=n} \frac{p^t}{q^t} = P_0 \cdot \frac{\left(q^n - p^n\right) \cdot p}{\left(q - p\right) \cdot q^n}$$

*) calculated with the Add-In *BWSesc* developed from the above formula

Ex_2.13_PV_Degradation

Item		Unit	Value
Given			
Revenues, 1st year	P_0	US$ /a	100,000
Degradation rate /factor	-j= 1.00% /a	-	1.010
Discount rate / factor	i= 8.60% /a	-	1.086
Life time		a	25
Results			
PV of revenues without degradation	j= 0.00% /a	US$	1,014,963
PV of revenues with degradation	-j= 1.00% /a	US$	929,274

$$PV_{n\_esc} = P_0 \cdot \sum_{t=1}^{t=n} \frac{p^t}{q^t} = P_0 \cdot \frac{\left(q^n - p^n\right) \cdot p}{\left(q - p\right) \cdot q^n}$$

*) calculated with the Add-In *BWSesc* developed from the above formula

EE-Ch-2_FinancialMaths_Examples.xls
Ex_2.14_Levleized O&M_costs

Item		Year	1	2	3	4	5	6	7	8	9	10
Nominal values		mln US$/a	0.0	5.5	6.5	25.0	11.5	13.0	14.0	35.0	18.0	20.0
PV factor	8.6 %/a	1.0860	1.09	1.18	1.28	1.39	1.51	1.64	1.78	1.93	2.10	2.28
Present values		86.53 mln US$	0.00	4.66	5.07	17.97	7.61	7.92	7.86	18.09	8.57	8.76
Levelized O&M		13.25 mln US$ /a	Discounted average									
in percent		1.5% CAPEX	CAPEX 900 mln									

or

MS-NPV = mln US$ 86.53

MS-PMT = mln US$/a 13.25

EE-Ch-2_FinancialMaths_Examples.xls
Ex_2.15_LevelizePersonnelCosts

Item		Unit	Value
Given			
Cost of personnel 1st year	P_0	US$ / (Pers. · a)	50,000
Life time of the project *"n"*	*n*	a	20
Escalation rate / factor	j = 4.000%	-	1.040
Discount rate /factor	i = 8.60%	-	1.086
Results			
Costs by the end of life time	*(1+j)^n x* P_0	US$ / (Pers. · a)	109,556
Levelized cost excl. escalation		US$ / a	50,000
Levelized cost incl. escalation *"P_{AN_esc}"*		US$ / a	69,693

*) Calculated with the Add-In function P_{AN_esc}, *see formula below*

$$P_{AN\_esc} = P_0 \cdot \frac{\left(q^n - p^n\right) \cdot p}{\left(q - p\right)} \times \frac{\left(q-1\right)}{q^n - 1} \qquad \left[\frac{\text{CU}}{\text{a}}\right]$$

*) calculated with the Add-In "*ANesc*" developed from the above formula

EE-Ch-2_FinancialMaths_Examples.xls
Ex_2.16_Levelize_crudeOil_price

Item	Unit	Values
Given		
Barrel price of crude oil in current US$	US$ / Bb	80
Discount rate *i*		8.60 %/a
Escalation rate *j* of crude oil	-	4.00 %/a
Period	years	20
Inflator *"p=1+j"*	-	1.0400
Discount factor *"q=1+q"*	-	1.086
Results		
Price at the end of the period "(1+j)^n×P_0	US$ / Bb	175
Levelized price during the life time *)	US$ / a	112

$$P_{AN\_esc} = PV_{n\_esc} \cdot \frac{\left(q^n - p^n\right) \cdot p}{\left(q - p\right)} \times \frac{\left(q-1\right)}{q^n - 1} \quad \left[\frac{\text{US\$}}{\text{a}}\right]$$

*) calculated with the Add-In "Anesc" developed from the above formula

Item	Equity	Loan
Asset shares	**30%**	**70%**
Risk free rate of return / interest	5.0 %/a	5.0 %/a
Venture risk premium	6.0 %/a	1.0 %/a
Country risk premium	0.0 %/a	0.0 %/a
Expected returnes, net	**11.0 %/a**	**6.0 %/a**
WACC$_n$ **in nominal terms, incl. tax**	**7.50 %/a**	
./. Expected Inflation rate	2.00 %/a	
WACC$_r$ **inflation adjusted**	**5.39 %/a**	

Item		Equity	Loan
Asset shares		**30%**	**70%**
Risk free rate of return / interest		5.0 %/a	5.0 %/a
venture risk premium		6.0 %/a	1.0 %/a
Country risk premium		0.0 %/a	0.0 %/a
Expected return after tax		**11.0 %/a**	**6.0 %/a**
Corporate tax *)	25%	3.7 %/a	0.0 %/a
Returns before tax, in nominal terms		14.7 %/a	6.0 %/a
***WACC$_n$* in nominal terms, incl. tax**		**8.60 %/a**	
./. Expected Inflation rate		2.00 %/a	
***WACC$_r$* inflation adjusted, incl. tax**		**6.47 %/a**	

*) **Note**: The tax rate is referred to the return on equity before tax. The calculation formula is therefore: tax = return on equity before tax x tax rate /(1 - tax rate)